Krylov Subspace Recycling With Randomized Sketching For Matrix Functions

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Abstract

I will discuss the importance of randomization in the development of *Krylov subspace recycling* algorithms for the efficient evaluation of a sequence of matrix function applications on a set of vectors [5]. Recycling methods are a special class of augmented Krylov subspace methods where the augmentation subspace for each problem is constructed or *recycled* from the Krylov subspace used to solve a previous problem in the sequence [4]. If selected appropriately, the presence of the recycled subspace can aid in accelerating the convergence of the iterative solver, thereby reducing the overall computational cost and runtime required to solve the full sequence of problems.

I will present the work in [1], where the *recycled Full Orthogonalization Method* (rFOM) for functions of matrices was shown to reduce the computational overhead and runtime required to evaluate a sequence of matrix function applications, when compared to the standard FOM approximation. I will discuss the development of rFOM, and show how it is not possible to develop a restarted implementation, resulting in excessive storage and orthogonalization costs as the number of iterations grows large.

As an alternative to restarts, I will introduce *sketched-recycled FOM (srFOM)*, which incorporates randomized sketching [2, 3] into rFOM in order to avoid excessive orthogonalization costs when working with non-Hermitian matrices. I will show results of numerical experiments which demonstrate the kind of performance gains we can achieve through sketching.

References

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