Birkhoff Averages, Invariant Sets, and Adaptive Filtering

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Abstract

In the design of magnetic confinement fusion devices (and many other applications), one is interested in classifying the trajectories of symplectic maps. That is, we consider discrete dynamical systems

$$\mathbf{x}_{t+1} = \mathbf{F}(\mathbf{x}_t)$$

where the map $\mathbf{F} : X \to X$ is symplectic. We are interested in classifying such trajectories as quasiperiodic orbits (invariant circles, islands) or as chaotic, and finding simple parameterizations of any quasiperiodic structures. In this talk, we describe a simple approach to these tasks by building a linear time-invariant system representation of the dynamics from a given starting point with a palindromic characteristic polynomial. This allows us to find a Fourier parameterization of invariant circles and islands from a single trajectory, as well as classifying trajectories as regular or chaotic. We connect our approach to ideas from extrapolation methods, adaptive filter design, and Birkhoff averages, and show examples of Birkhoff RRE on the standard map and magnetic field line dynamics.

References

[1] Maximillian Ruth and David Bindel. Finding Birkhoff averages via adaptive filtering. *Chaos*, 34(12):123109, December 2024.