Asynchronous methods meet randomized: Provable convergence rate

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Abstract

Asynchronous methods refer to parallel iterative procedures where each process performs its task without waiting for other processes to be completed, i.e., with whatever information it has locally available and with no synchronizations with other processes. For the numerical solution of a general partial differential equation on a domain, Schwarz iterative methods use a decomposition of the domain into two or more (usually overlapping) subdomains. In essence one is introducing new artificial boundary conditions Thus each process corresponds to a local solve with boundary conditions from the values in the neighboring subdomains.

Using this method as a solver, avoids the pitfall of synchronization required by the inner products in Krylov subspace methods. A scalable method results with either optimized Schwarz or when a coarse grid is added. Numerical results are presented on large three-dimensional problems illustrating the efficiency of asynchronous parallel implementations.

Most theorems show convergence of the asynchronous methods, but not a rate of convergence. In this talk, using the concepts of randomized linear algebra, we present provable convergence rate for the methods for a class of nonsymmetric linear systems. A key element in the new results is the choice of norm for which we can prove convergence of the residual in the expected value sense. Joint work with Andreas Frommer.