Some Modified Matrix Eigenvalue Problems

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Abstract

This is the title of well-known numerical linear algebra survey article by Gene Golub published in 1973 [1]. The article covers a range of matrix eigenvalue problems which require some manipulations before the standard algorithms may be used. I am using the same title to consider a new set of modified matrix eigenvalue problems. This includes constrained and bi-level optimizations arising from algorithms for fairness in machine learning, such as spectral clustering with group fairness [2] and fair principal component analysis [3]. We also consider eigenvalue optimization via 2D eigenvalue problem with applications to the calculation of the distance to instability among others [4], and stationary values of a quadratic form subject to non-homogeneous linear constraints for applications such as image segmentation with constraints [5]. I will discuss how to explore the underlying structures of these problems to turn them into our familiar eigenvalue problems and algorithms. This talk is based on joint work with Ian Davidson, Aaron Davis, Ren-Cang Li, Ding Lu, Tianyi Lu, Junhui Shen, Yangfeng Su, Ji Wang, and Yunshen Zhou.

References

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